# Dynamic Programming for Parsing and Estimation of Stochastic Unification-Based Grammars 

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## Talk outline

- Stochastic Unification-Based Grammars (SUBGs)
- Parsing and estimation of SUBGs
- Avoiding enumerating all parses
- Maxwell and Kaplan (1995) packed parse representations
- Feature locality
- Parse weight is a product of functions of parse fragment weights
- Graphical model calculation of argmax/sum

Related work: Miyao and Tsuji (2002) "Maximum Entropy Estimation for Feature Forests" HLT

## Lexical-Functional Grammar (UBG)




## Stochastic Unification-based Grammars

- A unification-based grammar defines a set of possible parses $\mathcal{Y}(w)$ for each sentence $w$.
- Features $f_{1}, \ldots, f_{m}$ are real-valued functions on parses
- Attachment location (high, low, argument, adjunct, etc.)
- Head-to-head dependencies
- Probability defined by conditional log-linear model

$$
\begin{aligned}
W(y) & =\exp \left(\sum_{j=1}^{m} \lambda_{j} f_{j}(y)\right)=\prod_{j=1}^{m} \theta_{j}^{f_{j}(y)} \\
\operatorname{Pr}(y \mid w) & =W(y) / Z(w)
\end{aligned}
$$

where $\theta_{j}=e^{\lambda_{j}}>0$ are feature weights and
$Z(w)=\sum_{y \in \mathcal{Y}(w)} W(y)$ is the partition function.
Johnson et al (1999) "Parsing and estimation for SUBGs", Proc ACL

## Estimating feature weights

- Several algorithms for maximum conditional likelihood estimation
- Various iterative scaling algorithms
- Conjugate gradient and other optimization algorithms
- These algorithms are iterative
$\Rightarrow$ repeated reparsing of training data
- All of these algorithms require conditional expectations

$$
E\left[f_{j} \mid w\right]=\sum_{y \in \mathcal{Y}(w)} f_{j}(y) \operatorname{Pr}(y \mid w)
$$

- Can we calculate these statistics and find the most likely parse without enumerating all parses $\mathcal{Y}(w)$ ? YES *


## Maxwell and Kaplan packed parses

- A parse $y$ consists of set of fragments $\xi \in y$ (MK algorithm)
- A fragment is in a parse when its context function is true
- Context functions are functions of context variables $X_{1}, X_{2}, \ldots$
- The variable assignment must satisfy "not no-good" functions
- Each parse is identified by a unique context variable assignment

$$
\begin{aligned}
& \xi=\text { "the cat on the mat" } \\
& \xi_{1}=\text { "with a hat" } \\
X_{1} \rightarrow & \text { "attach } D \text { to } B \text { " } \\
\neg X_{1} \rightarrow & \text { "attach } D \text { to } A "
\end{aligned}
$$



## Feature locality

- Features must be local to fragments: $f_{j}(y)=\sum_{\xi \in y} f_{j}(\xi)$
- May require changes to UBG to make all features local

$$
\begin{aligned}
& \xi=\text { "the cat on the mat" } \\
& \xi_{1}=\text { "with a hat" } \\
X_{1} \rightarrow & \text { "attach } D \text { to } B " \wedge\left(\xi_{1} \text { ATTACH }\right)=\mathrm{LOW} \\
\neg X_{1} \rightarrow & \text { "attach } D \text { to } A " \wedge\left(\xi_{1} \text { ATTACH }\right)=\mathrm{HIGH}
\end{aligned}
$$



## Feature locality decomposes $W(y)$

- Feature locality: the weight of a parse is the product of weights of its fragments

$$
\begin{aligned}
W(y) & =\prod_{\xi \in y} W(\xi), \quad \text { where } \\
W(\xi) & =\prod_{j=1}^{m} \theta_{j}^{f_{j}(\xi)}
\end{aligned}
$$

$$
\begin{aligned}
& W(\xi=\text { "the cat on the mat" }) \\
& W\left(\xi_{1}=\text { "with a hat" }\right) \\
X_{1} \rightarrow & W\left(\text { "attach } D \text { to } B^{\prime \prime} \wedge\left(\xi_{1} \text { ATTACH }\right)=\text { LOW }\right) \\
\neg X_{1} \rightarrow & W\left(\text { "attach } D \text { to } A " \wedge\left(\xi_{1} \text { ATTACH }\right)=\text { HIGH }\right)
\end{aligned}
$$

## Not No-goods

- "Not no-goods" identify the variable assignments that correspond to parses

$$
\begin{aligned}
& \xi=\text { "I read a book" } \\
& \xi_{1}=\text { "on the table" } \\
X_{1} \wedge X_{2} \rightarrow & \text { "attach } D \text { to } B^{\prime \prime} \\
X_{1} \wedge \neg X_{2} \rightarrow & \text { "attach } D \text { to } A^{\prime \prime} \\
\neg X_{1} \rightarrow & \text { "attach } D \text { to } C^{\prime \prime} \\
& X_{1} \vee X_{2}
\end{aligned}
$$

## Identify parses with variable assignments

- Each variable assignment uniquely identifies a parse
- For a given sentence $w$, let $W^{\prime}(x)=W(y)$ where $y$ is the parse identified by $x$
$\Rightarrow$ Argmax/sum/expectations over parses can be computed over context variables instead

Most likely parse: $\hat{x}=\operatorname{argmax}_{x} W^{\prime}(x)$
Partition function: $Z(w)=\sum_{x} W^{\prime}(x)$
Expectation: ${ }^{\star} E\left[f_{j} \mid w\right]=\sum_{x} f_{j}(x) W^{\prime}(x) / Z(w)$

## $W^{\prime}$ is a product of functions of $X$

- Then $W^{\prime}(X)=\prod_{A \in \mathcal{A}} A(X)$, where:
- Each line $\alpha(X) \rightarrow \xi$ introduces a term $W(\xi)^{\alpha(X)}$
- A "not no-good" $\eta(X)$ introduces a term $\eta(X)$

$$
\begin{array}{rccc}
\alpha(X) & \rightarrow & \xi & \times \\
\vdots & & W(\xi)^{\alpha(X)} \\
& \eta(X) & \times & \vdots \\
\vdots & & \times & \eta(X) \\
& & \vdots
\end{array}
$$

$\Rightarrow W^{\prime}$ is a Markov Random Field over the context variables $X$

## $W^{\prime}$ is a product of functions of $X$

$$
\begin{aligned}
W^{\prime}\left(X_{1}\right)= & W(\xi=\text { "the cat on the mat" }) \\
\times & W\left(\xi_{1}=\text { "with a hat" }\right) \\
\times & W\left(\text { "attach } D \text { to } B " \wedge\left(\xi_{1} \text { ATTACH }\right)=\mathrm{LOW}\right)^{X_{1}} \\
\times & W\left(\text { "attach } D \text { to } A " \wedge\left(\xi_{1} \text { ATTACH }\right)=\mathrm{HIGH}\right)^{\neg X_{1}}
\end{aligned}
$$



## Product expressions and graphical models

- MRFs are products of terms, each of which is a function of (a few) variables
- Graphical models provide dynamic programming algorithms for Markov Random Fields (MRF) (Pearl 1988)
- These algorithms implicitly factorize the product
- They generalize the Viterbi and Forward-Backward algorithms to arbitrary graphs (Smyth 1997)
$\Rightarrow$ Graphical models provide dynamic programming techniques for parsing and training Stochastic UBGs


## Factorization example

$$
\begin{aligned}
W^{\prime}\left(X_{1}\right)= & W(\xi=\text { "the cat on the mat" }) \\
\times & W\left(\xi_{1}=\text { "with a hat" }\right) \\
\times & W\left(\text { "attach } D \text { to } B^{\prime \prime} \wedge\left(\xi_{1} \text { ATTACH }\right)=\text { LOW }\right)^{X_{1}} \\
\times & W\left(\text { "attach } D \text { to } A^{\prime \prime} \wedge\left(\xi_{1} \text { ATTACH }\right)=\text { HIGH }\right)^{\neg X_{1}}
\end{aligned}
$$

$$
\begin{aligned}
\max _{X_{1}} W^{\prime}\left(X_{1}\right)= & W(\xi=\text { "the cat on the mat" }) \\
\times & W\left(\xi_{1}=\text { "with a hat" }\right) \\
\times & \max _{X_{1}}\left(\begin{array}{l}
W\left(\text { "attach } D \text { to } B^{\prime \prime} \wedge\left(\xi_{1} \text { ATTACH }\right)=\text { LOW }\right)^{X_{1}}, \\
W\left(\text { "attach } D \text { to } A " \wedge\left(\xi_{1} \text { ATTACH }\right)=\text { HIGH }\right)^{\neg X_{1}}
\end{array}\right.
\end{aligned}
$$

## Dependency structure graph $G_{\mathcal{A}}$

$$
Z(w)=\sum_{x} W^{\prime}(x)=\sum_{x} \prod_{A \in \mathcal{A}} A(x)
$$

- $G_{\mathcal{A}}$ is the dependency graph for $\mathcal{A}$
- context variables $X$ are vertices of $G_{\mathcal{A}}$
- $G_{\mathcal{A}}$ has an edge $\left(X_{i}, X_{j}\right)$ if both are arguments of some $A \in \mathcal{A}$

$$
A(X)=a\left(X_{1}, X_{3}\right) b\left(X_{2}, X_{4}\right) c\left(X_{3}, X_{4}, X_{5}\right) d\left(X_{4}, X_{5}\right) e\left(X_{6}, X_{7}\right)
$$



## Graphical model computations

$$
\begin{aligned}
Z & =\sum_{x} a\left(x_{1}, x_{3}\right) b\left(x_{2}, x_{4}\right) c\left(x_{3}, x_{4}, x_{5}\right) d\left(x_{4}, x_{5}\right) e\left(x_{6}, x_{7}\right) \\
Z_{1}\left(x_{3}\right) & =\sum_{x_{1}} a\left(x_{1}, x_{3}\right) \\
Z_{2}\left(x_{4}\right) & =\sum_{x_{2}} b\left(x_{2}, x_{4}\right) \\
Z_{3}\left(x_{4}, x_{5}\right) & =\sum_{x_{3}} c\left(x_{3}, x_{4}, x_{5}\right) Z_{1}\left(x_{3}\right) \\
Z_{4}\left(x_{5}\right) & =\sum_{x_{4}} d\left(x_{4}, x_{5}\right) Z_{2}\left(x_{4}\right) Z_{3}\left(x_{4}, x_{5}\right) \\
Z_{5} & =\sum_{x_{5}} Z_{4}\left(x_{5}\right) \\
Z_{6}\left(x_{7}\right) & =\sum_{x_{6}} e\left(x_{6}, x_{7}\right) \\
Z_{7} & =\sum_{x_{7}} Z_{6}\left(x_{7}\right) \\
Z & =Z_{5} Z_{7} \\
& =\left(\sum_{x_{5}} Z_{4}\left(x_{5}\right)\right)\left(\sum_{x_{7}} Z_{6}\left(x_{7}\right)\right)
\end{aligned}
$$

See: Pearl (1988) Probabilistic Reasoning in Intelligent Systems

## Graphical model for Homecentre example

Use a damp, lint-free cloth to wipe the dust and dirt buildup from the scanner plastic window and rollers.


## Computational complexity

- Polynomial in $m=$ the maximum number of variables in the dynamic programming functions $\geq$ the number of variables in any function $A$
- $m$ depends on the ordering of variables (and $G$ )
- Finding the variable ordering that minimizes $m$ is NP-complete, but there are good heuristics
$\Rightarrow$ Worst case exponential (no better than enumerating the parses), but average case might be much better
- Much like UBG parsing complexity


## Conclusion

- There are DP algorithms for parsing and estimation from packed parses that avoid enumerating parses
- Generalizes to all Truth Maintenance Systems (not grammar specific)
- Features must be local to parse fragments
- May require adding features to the grammar
- Worst-case exponential complexity; average case?
- Makes available techniques for graphical models to packed parse representations
- MCMC and other sampling techniques


## Future directions

- Reformulate "hard" grammatical constraints as "soft" stochastic features
- Underlying grammar permits all possible structural combinations
- Grammatical constraints reformulated as stochastic features
- Is this computation tractable?
- Comparison with Miyao and Tsujii (2002)

