# An introduction to grammars and parsing 

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## Talk outline

- What is computational linguistics?
- Probabilistic grammars
- Identifying phrase structure (parsing)
- Learning phrase structure
- More realistic grammars


## What is computational linguistics?

Computational linguistics studies the computational processes involved in language production, comprehension and acquisition.

- assumption that language is inherently computational
- scientific side:
- modeling human performance (computational psycholinguistics)
- understanding how it can be done at all
- technological applications:
- speech recognition
- information extraction (who did what to whom) and question answering
- machine translation (translation by computer)


## (Some of the) problems in modeling language

+ Language is a product of the human mind $\Rightarrow$ any structure we observe is a product of the mind
- Language involves a transduction between form and meaning, but we don't know much about the way meanings are represented
$+/-$ We have (reasonable?) guesses about some of the computational processes involved in language
- We don't know very much about the cognitive processes that language interacts with
- We know little about the anatomical layout of language in the brain
- We know little about neural networks that might support linguistic computations


## Aspects of linguistic structure

- Phonetics: the (production and perception) of speech sounds
- Phonology: the organization and regularities of speech sounds
- Morphology: the structure and organization of words
- Syntax: the way words combine to form phrases and sentences
- Semantics: the way meaning is associated with sentences
- Pragmatics: how language can be used to do things

In general the further we get from speech, the less well we understand what's going on!

## Aspects of syntactic and semantic structure




- Anaphora: it refers to baked beans
- Predicate-argument structure: the students is agent of eat
- Discourse structure: second clause is contrasted with first

These all refer to phrase structure entities! Parsing is the process of recovering these entities.

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## Context free grammars

A context-free grammar $G=(\mathcal{V}, \mathcal{S}, s, \mathcal{R})$ consists of:

- $\mathcal{V}$, a finite set of terminals $\left(\mathcal{V}_{0}=\{\right.$ Sam, Sasha, thinks, snores $\left.\}\right)$
- $\mathcal{S}$, a finite set of non-terminals disjoint from $\mathcal{V}\left(\mathcal{S}_{0}=\{\mathrm{S}, \mathrm{NP}, \mathrm{VP}, \mathrm{V}\}\right)$
- $\mathcal{R}$, a finite set of productions of the form $A \rightarrow X_{1} \ldots X_{n}$, where $A \in \mathcal{S}$ and each $X_{i} \in \mathcal{S} \cup \mathcal{V}$
- $s \in \mathcal{S}$ is called the start symbol $\left(s_{0}=\mathrm{S}\right)$
$G$ generates a tree $\psi$ iff
- The label of $\psi$ 's root node is $s$
- For all local trees with parent $A$ and children $X_{1} \ldots X_{n}$ in $\psi$ $A \rightarrow X_{1} \ldots X_{n} \in \mathcal{R}$
$G$ generates a string $w \in \mathcal{V}^{\star}$ iff $w$ is the terminal yield of a tree generated by $G$


Productions
$\mathrm{S} \rightarrow \mathrm{NP}$ VP
$N P \rightarrow$ Sam
$N P \rightarrow$ Sasha
$\mathrm{VP} \rightarrow \mathrm{V}$
$\mathrm{VP} \rightarrow \mathrm{V}$ S
$V \rightarrow$ thinks
snores $V \rightarrow$ snores

## CFGs as "plugging" systems



## "Pluggings"



Resulting tree

- Goal: no unconnected "sockets" or "plugs"
- The productions specify available types of components
- In a probabilistic CFG each type of component has a "price"


## Structural Ambiguity

- CFGs can capture structural ambiguity in language.

- Ambiguity generally grows exponentially in the length of the string.
- The number of ways of parenthesizing a string of length $n$ is Catalan ( $n$ )
- Broad-coverage statistical grammars are astronomically ambiguous.


## Derivations

A CFG $G=(\mathcal{V}, \mathcal{S}, s, \mathcal{R})$ induces a rewriting relation $\Rightarrow_{G}$, where $\gamma A \delta \Rightarrow_{G} \gamma \beta \delta$ iff $A \rightarrow \beta \in \mathcal{R}$ and $\gamma, \delta \in(\mathcal{S} \cup \mathcal{V})^{\star}$.

A derivation of a string $w \in \mathcal{V}^{\star}$ is a finite sequence of rewritings $s \Rightarrow_{G} \ldots \Rightarrow_{G} w . \quad \Rightarrow_{G}^{\star}$ is the reflexive and transitive closure of $\Rightarrow_{G}$.

The language generated by $G$ is $\left\{w: s \Rightarrow^{\star} w, w \in \mathcal{V}^{\star}\right\}$.
$G_{0}=\left(\mathcal{V}_{0}, \mathcal{S}_{0}, \mathrm{~S}, \mathcal{R}_{0}\right), \mathcal{V}_{0}=\{\mathrm{Sam}$, Sasha, likes, hates $\}, \mathcal{S}_{0}=\{\mathrm{S}, \mathrm{NP}, \mathrm{VP}, \mathrm{V}\}$, $\mathcal{R}_{0}=\{\mathrm{S} \rightarrow \mathrm{NP} \mathrm{VP}, \mathrm{VP} \rightarrow \mathrm{VNP}, \mathrm{NP} \rightarrow$ Sam, NP $\rightarrow$ Sasha, $\mathrm{V} \rightarrow$ likes, $\mathrm{V} \rightarrow$ hates $\}$ S
$\Rightarrow$ NP VP $\quad$ Steps in a terminating
$\Rightarrow$ NP V NP
$\Rightarrow$ Sam V NP
$\Rightarrow$ Sam V Sasha Left-most and right-most
$\Rightarrow$ Sam likes Sasha derivations are unique

likes

## Probabilistic Context Free Grammars

A Probabilistic Context Free Grammar (PCFG) $G$ consists of

- a $\operatorname{CFG}(\mathcal{V}, \mathcal{S}, S, \mathcal{R})$ with no useless productions, and
- production probabilities $p(A \rightarrow \beta)=\mathrm{P}(\beta \mid A)$ for each $A \rightarrow \beta \in \mathcal{R}$, the conditional probability of an $A$ expanding to $\beta$

A production $A \rightarrow \beta$ is useless iff it is not used in any terminating derivation, i.e., there are no derivations of the form $S \Rightarrow^{\star} \gamma A \delta \Rightarrow \gamma \beta \delta \Rightarrow^{*} w$ for any $\gamma, \delta \in(\mathcal{V} \cup \mathcal{S})^{\star}$ and $w \in \mathcal{V}^{\star}$.

If $r_{1} \ldots r_{n}$ is a sequence of productions used to generate a tree $\psi$, then

$$
\begin{aligned}
\mathrm{P}_{G}(\psi) & =p\left(r_{1}\right) \ldots p\left(r_{n}\right) \\
& =\prod_{r \in \mathcal{R}} p(r)^{f_{r}(\psi)}
\end{aligned}
$$

where $f_{r}(\psi)$ is the number of times $r$ is used in deriving $\psi$
$\sum_{\psi} \mathrm{P}_{G}(\psi)=1$ if $p$ satisfies suitable constraints

## Example PCFG



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## Computing the probability of a tree

$$
\begin{array}{ll}
1.0 & \mathrm{~S} \rightarrow \mathrm{NP} \text { VP } \\
0.75 & \mathrm{NP} \rightarrow \text { George } \\
0.6 & \mathrm{~V} \rightarrow \text { barks }
\end{array}
$$


$1.0 \quad \mathrm{VP} \rightarrow \mathrm{V}$
$0.25 \mathrm{NP} \rightarrow \mathrm{Al}$
$0.4 \mathrm{~V} \rightarrow$ snores

$\mathrm{P}\left(\right.$| S |  |
| :---: | :---: |
| NP | VP |
| I | I |
| Al | V |
|  |  |
|  |  |
|  | snores |$)=0.1$

## Things we want to compute

1. What is the probability $\mathrm{P}_{G}(w)$ of the string $w$ ? (language modeling)

$$
\mathrm{P}_{G}(w)=\mathrm{P}_{G}\left(s \Rightarrow^{*} w\right)=\sum_{\psi \in \Psi_{G}(w)} \mathrm{P}_{G}(\psi)
$$

2. What is the most probable parse $\hat{\psi}(w)$ of a string $w$ ? (parsing)

$$
\hat{\psi}(w)=\underset{\psi \in \Psi_{G}(w)}{\operatorname{argmax}} \mathrm{P}_{G}(\psi)
$$

where:
$\Psi_{G}(w)$ is the set of parse trees for $w$ generated by $G$, and
$\mathrm{P}_{G}(\psi)$ is the probability of tree $\psi$ wrt grammar $G$.
Naive algorithm:

1. Compute set of parse trees $\Psi_{G}(w)$ for $w$
2. Take max/sum as appropriate

## String positions

String positions are a systematic way of representing substrings in a string.
A string position of a string $w=x_{0} \ldots x_{n-1}$ is an integer $0 \leq i \leq n$.
A substring of $w$ is represented by a pair $(i, j)$ of string positions, where $0 \leq i \leq j \leq n$.
$w_{i, j}$ represents the substring $w_{i} \ldots w_{j-1}$


Example:
$w_{0,1}=$ Howard, $w_{1,3}=$ likes mangoes, $w_{0,0}=w_{1,1}=w_{2,2}=w_{3,3}=\epsilon$

- Nothing depends on string positions being numbers, so
- this all generalizes to speech recognizer lattices, which are graphs where vertices correspond to word boundaries



## PCFG "Inside" algorithm

The inside algorithm for computing the probability of a string is a generalization of the backward algorithm.

Assume $G=(\mathcal{V}, \mathcal{S}, s, \mathcal{R}, p)$ is in Chomsky Normal Form, i.e., all productions are of the form $A \rightarrow B C$ or $A \rightarrow x$, where $A, B, C \in \mathcal{S}, x \in \mathcal{V}$.
Goal: To compute $\mathrm{P}(w)=\sum_{\psi \in \Psi_{G}(w)} \mathrm{P}(\psi)=\mathrm{P}\left(s \Rightarrow^{\star} w\right)$
Data structure: A table $\mathrm{P}\left(A \Rightarrow^{\star} w_{i, j}\right)$ for $A \in \mathcal{S}$ and $0 \leq i<j \leq n$
Base case: $\mathrm{P}\left(A \Rightarrow^{\star} w_{i, i+1}\right)=p\left(A \rightarrow w_{i}\right)$ for $i=0, \ldots, n-1$
Recursion: $\mathrm{P}\left(A \Rightarrow^{\star} w_{i, k}\right)$

$$
=\sum_{j=i+1}^{k-1} \sum_{A \rightarrow B} p(A \rightarrow B C) \mathrm{P}\left(B \Rightarrow^{\star} w_{i, j}\right) \mathrm{P}\left(C \Rightarrow^{\star} w_{j, k}\right)
$$

Return: $\mathrm{P}\left(s \Rightarrow^{\star} w_{0, n}\right)$

## Dynamic programming recursion

$$
\mathrm{P}_{G}\left(A \Rightarrow^{\star} w_{i, k}\right)=\sum_{j=i+1}^{k-1} \sum_{A \rightarrow B C \in \mathcal{R}(A)} p(A \rightarrow B C) \mathrm{P}_{G}\left(B \Rightarrow^{\star} w_{i, j}\right) \mathrm{P}_{G}\left(C \Rightarrow^{\star} w_{j, k}\right)
$$


$\mathrm{P}_{G}\left(A \Rightarrow^{\star} w_{i, k}\right)$ is called an "inside probability".

## Example PCFG parse

| 1.0 | $\mathrm{~S} \rightarrow \mathrm{NP} \mathrm{VP}$ | 1.0 |
| :--- | :--- | :--- |
| $\mathrm{VP} \rightarrow \mathrm{V} \mathrm{NP}$ |  |  |
| 0.7 | $\mathrm{NP} \rightarrow$ George | 0.3 |
| $\mathrm{NP} \rightarrow$ John |  |  |
| 0.5 | $\mathrm{~V} \rightarrow$ likes | 0.5 |

Right string position
1
2
3


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## Two approaches to computational linguistics

"Rationalist": Linguist formulates generalizations and expresses them in a grammar
"Empiricist": Collect a corpus of examples, linguists annotate them with relevant information, a machine learning algorithm extracts generalizations

- I don't think there's a deep philosophical difference here, but many people do
- Continuous models do much better than categorical models (statistical inference uses more information than categorical inference)
- Humans are lousy at estimating numerical probabilities, but luckily parameter estimation is the one kind of machine learning that (sort of) works


## Treebanks, prop-banks and discourse banks

- A treebank is a corpus of phrase structure trees
- The Penn treebank consists of about a million words from the Wall Street Journal, or about 40,000 trees.
- The Switchboard corpus consists of about a million words of treebanked spontaneous conversations, linked up with the acoustic signal.
- Treebanks are being constructed for other languages also
- The Penn treebank is being annotated with predicate argument structure (PropBank) and discourse relations.


## Estimating PCFGs from visible data



Rule
$S \rightarrow N P V P$
$\mathrm{NP} \rightarrow$ rice
Count Rel Freq

| $\mathrm{NP} \rightarrow$ corn | 1 | $1 / 3$ |
| :--- | :--- | :---: |
| $\mathrm{VP} \rightarrow$ grows | 3 | 1 |

1
2/3
$1 / 3$
1
$P(\overbrace{\substack{\mathrm{VPP} \\ \text { rice } \\ \text { grows }}}^{\mathrm{S}})=2 / 3$

(The relative frequency estimator is also the MLE for PFSA, of course).

## Properties of MLE

The relative frequency estimator is the MLE for PCFGS, so it has the following properties:

- Consistency: As the sample size grows, the estimates of the parameters converge on the true parameters
- Asymptotic optimality: For large samples, there is no other consistent estimator whose estimates have lower variance
- Sparse data is the big problem with the MLE.
- Rules that do not occur in the training data get zero probability
- Aside from this, MLEs for statistical grammars work well in practice.
- The Penn Treebank has $\approx 1.2$ million words of Wall Street Journal text annotated with syntactic trees
- The PCFG estimated from the Penn Treebank has $\approx 15,000$ rules


## Unsupervised training and EM

Expectation Maximization (EM) is a general technique for approximating the MLE when estimating parameters $p$ from the visible data $x$ is difficult, but estimating $p$ from augmented data $z=(x, y)$ is easier ( $y$ is the hidden data).

## The EM algorithm given visible data $x$ :

1. guess initial value $p_{0}$ of parameters (e.g., rule probabilities)
2. repeat for $i=0,1, \ldots$ until convergence:

Expectation step: For all $y_{1}, \ldots, y_{n} \in \mathcal{Y}$, generate pseudo-data $\left(x, y_{1}\right), \ldots,\left(x, y_{n}\right)$, where $(x, y)$ has "frequency" $\mathrm{P}_{p_{i}}(y \mid x)$

Maximization step: Set $p_{i+1}$ to the MLE from the pseudo-data
The EM algorithm finds the MLE $\hat{p}(x)=L_{x}(p)$ of the visible data $x$.
Sometimes it is not necessary to explicitly generate the pseudo-data $(x, y)$; often it is possible to perform the maximization step directly from sufficient statistics (for PCFGs, the expected production frequencies)

## Dynamic programming for $E_{G}\left[n_{A \rightarrow B C} \mid w\right]$

$$
E_{G}\left[n_{A \rightarrow B C} \mid w\right]=\sum_{0 \leq i<j<k \leq n} E_{G}\left[A_{i, k} \rightarrow B_{i, j} C_{j, k} \mid w\right]
$$

The expected fraction of parses of $w$ in which $A_{i, k}$ rewrites as $B_{i, j} C_{j, k}$ is:

$$
\begin{aligned}
& E_{G}\left[A_{i, k} \rightarrow B_{i, j} C_{j, k} \mid w\right] \\
& \qquad=\frac{\mathrm{P}\left(S \Rightarrow^{\star} w_{1, i} A w_{k, n}\right) p(A \rightarrow B C) \mathrm{P}\left(B \Rightarrow^{\star} w_{i, j}\right) \mathrm{P}\left(C \Rightarrow^{\star} w_{j, k}\right)}{\mathrm{P}_{G}(w)} \\
& w_{0, i}
\end{aligned}
$$

## Calculating $\mathrm{P}_{G}\left(S \Rightarrow^{\star} w_{0, i} A w_{k, n}\right)$

Known as "outside probabilities" (but if $G$ contains unary productions, they can be greater than 1).

Recursion from larger to smaller substrings in $w$.
Base case: $\mathrm{P}\left(S \Rightarrow^{\star} w_{0,0} S w_{n, n}\right)=1$
Recursion: $\mathrm{P}\left(S \Rightarrow^{\star} w_{0, j} C w_{k, n}\right)=$

$$
\begin{aligned}
& \sum_{i=0}^{j-1} \sum_{\substack{A, B \in \mathcal{S} \\
A \rightarrow B C \in \mathcal{R}}} \mathrm{P}\left(S \Rightarrow^{\star} w_{0, i} A w_{k, n}\right) p(A \rightarrow B C) \mathrm{P}\left(B \Rightarrow^{\star} w_{i, j}\right) \\
+ & \sum_{l=k+1}^{n} \sum_{\substack{A, D \in \mathcal{S} \\
A \rightarrow C \\
D \in \mathcal{R}}} \mathrm{P}\left(S \Rightarrow^{\star} w_{0, j} A w_{l, n}\right) p(A \rightarrow C D) \mathrm{P}\left(D \Rightarrow^{\star} w_{k, l}\right)
\end{aligned}
$$

## Recursion in $\mathrm{P}_{G}\left(S \Rightarrow^{\star} w_{0, i} A w_{k, n}\right)$



## The EM algorithm for PCFGs

Infer hidden structure by maximizing likelihood of visible data:

1. guess initial rule probabilities
2. repeat until convergence
(a) parse a sample of sentences
(b) weight each parse by its conditional probability
(c) count rules used in each weighted parse, and estimate rule frequencies from these counts as before

EM optimizes the marginal likelihood of the strings $D=\left(w_{1}, \ldots, w_{n}\right)$
Each iteration is guaranteed not to decrease the likelihood of $D$, but EM can get trapped in local minima.

The Inside-Outside algorithm can produce the expected counts without enumerating all parses of $D$.

When used with PFSA, the Inside-Outside algorithm is called the Forward-Backward algorithm (Inside=Backward, Outside=Forward)

## Example: The EM algorithm with a toy PCFG



Initial rule probs rule
$\mathrm{VP} \rightarrow \mathrm{V} \quad 0.2$
$\mathrm{VP} \rightarrow \mathrm{V}$ NP $\quad 0.2$
$\mathrm{VP} \rightarrow \mathrm{NP}$ V $\quad 0.2$
$\mathrm{VP} \rightarrow \mathrm{V}$ NP NP 0.2
$\mathrm{VP} \rightarrow$ NP NP V 0.2

Det $\rightarrow$ the $\quad 0.1$
$\mathrm{N} \rightarrow$ the $\quad 0.1$
$\mathrm{V} \rightarrow$ the $\quad 0.1$
"English" input
the dog bites
the dog bites a man
a man gives the dog a bone
..
"pseudo-Japanese" input the dog bites
the dog a man bites
a man the dog a bone gives
...

## Probability of "English"



## Rule probabilities from "English"



## Probability of "Japanese"



## Rule probabilities from"Japanese"



## Learning in statistical paradigm

- The likelihood is a differentiable function of rule probabilities $\Rightarrow$ learning can involve small, incremental updates
- Learning new structure (rules) is hard, but ...
- Parameter estimation can approximate rule learning
- start with "superset" grammar
- estimate rule probabilities
- discard low probability rules


## Applying EM to real data

- ATIS treebank consists of 1,300 hand-constructed parse trees
- ignore the words (in this experiment)
- about 1,000 PCFG rules are needed to build these trees



## Experiments with EM

1. Extract productions from trees and estimate probabilities probabilities from trees to produce PCFG.
2. Initialize EM with the treebank grammar and MLE probabilities
3. Apply EM (to strings alone) to re-estimate production probabilities.
4. At each iteration:

- Measure the likelihood of the training data and the quality of the parses produced by each grammar.
- Test on training data (so poor performance is not due to overlearning).


## Likelihood of training strings



## Quality of ML parses



## Why does EM do so poorly?

- EM assigns trees to strings to maximize the marginal probability of the strings, but the trees weren't designed with that in mind
- We have an "intended interpretation" of categories like NP, VP, etc., which EM has no way of knowing
- Our grammar models are defective; real languages aren't context-free
- How can information about $\mathrm{P}(w)$ provide information about $\mathrm{P}(\psi \mid w)$ ?
- ... but no one really knows.


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## Subcategorization

Grammars that merely relate categories miss a lot of important linguistic relationships.

$$
R_{3}=\{\mathrm{VP} \rightarrow \mathrm{~V}, \mathrm{VP} \rightarrow \mathrm{VNP}, \mathrm{~V} \rightarrow \text { sleeps }, \mathrm{V} \rightarrow \text { likes }, \ldots\}
$$



Verbs and other heads of phrases subcategorize for the number and kind of complement phrases they can appear with.

## CFG account of subcategorization

General idea: Split the preterminal states to encode subcategorization.

$R_{4}=\left\{\mathrm{VP} \rightarrow \underset{\left.{ }_{[ }\right]}{\mathrm{V}}, \mathrm{VP} \rightarrow \underset{{ }_{[-}}{\mathrm{V} P]} \mathrm{NP}, \underset{\left[\_\right]}{\mathrm{V}} \rightarrow\right.$ sleeps, $\underset{[-\mathrm{NP}]}{\mathrm{V}} \rightarrow$ likes, $\left.\ldots\right\}$
The "split preterminal states" restrict which contexts verbs can appear in.
Sparse data becomes a big problem; addressed with regularization and smoothing.

## Selectional preferences

Head-to-head dependencies are an approximation to real-world knowledge.


But note that selectional preferences involve more than head-to-head dependencies

Al drives a (\#toy model) car

## Head to head dependencies and bilexical rules



The number of possible rules grows rapidly; with a $\approx 10^{4}$ word vocabulary one might expect over $10^{8}$ possible bilexical rules.

Sparse data is the big problem with grammars like this.

## Nonlocal "movement" dependencies



Subcategorization and selectional preferences are preserved under movement.

Movement can be encoded using recursive nonterminals (unification grammars).

## Nonterminal labels as communication channels

The nonterminal label is the communication channel between two parts of the tree.

Learn or design the structure of non-terminals so they pass the appropriate information around the tree.

_ WH movement dependency
Modern statistical parsers pass around 7 different features through the tree, and condition productions on them.
Sparse data and therefore smoothing are major issues!

## Summary and Conclusion

- Computational linguistics is great fun!
- ... and maybe will help us understand deep things about language and the mind
- Language involves rich compositional structure
- ... and grammars are a way of describing that structure
- Probabilistic grammars give us a systematic way of distinguishing more likely structures from less likely structures
- The number of parses (structures) can grow exponentially with sentence length
- ... but there are polynomial-time dynamic programming algorithms for most of the important problems
- Sparse data is a big problem for learning realistic grammars


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## Applications of neural networks in CL

- The big problem: how can NNs represent compositional structure?
- NNs are sometimes used as components in larger models (e.g., predicting a node's label given the labels of its children)
- other machine learning techniques often work better, and are computationally more efficient
- Applying recurrent $N N$ models of structure in time has generally not been successful ("a dog walking on its hind legs")
- Recurrent NNs seem to be equivalent to (factored) finite state machines
$\Rightarrow$ incapable of capturing two-dimensional nature of phrase structure


## "Brute force" NN simulation of CKY parsing

$$
\mathrm{P}_{G}\left(A \Rightarrow^{\star} w_{i, k}\right)=\sum_{j=i+1}^{k-1} \sum_{A \rightarrow B C \in \mathcal{R}(A)} p(A \rightarrow B C) \mathrm{P}_{G}\left(B \Rightarrow^{\star} w_{i, j}\right) \mathrm{P}_{G}\left(C \Rightarrow^{\star} w_{j, k}\right)
$$



$$
w_{i, j} \quad w_{j, k}
$$

- Introduce a node for each $A_{i, k}$ and for each ( $B_{i, j}, C_{j, k}$ )
- $\left(B_{i, j}, C_{j, k}\right)$ computes $p(A \rightarrow B C) \mathrm{P}_{G}\left(B \Rightarrow^{\star} w_{i, j}\right) \mathrm{P}_{G}\left(C \Rightarrow^{\star} w_{j, k}\right)$
- $A_{i, k}$ sums over all $A \rightarrow B C$ and $\left(B_{i, j}, C_{j, k}\right)$
- Asynchronous (non-real-time) model with unnatural parameter tying


## Incremental parsing algorithms

- Parsing involves incrementally constructing the parse tree
- We require that words are incorporated in left-to-right order
- Because of the two-dimensional nature of phrase structure trees, there are many ways of parsing incrementally

Top-down: Posit parent before any children
Bottom-up: Posit parent after all children
Left-corner: Posit parent after first child

- Every incremental parsing algorithm corresponds to different sets of cuts through edges in the phrase structure tree
- Only the edges on such cuts are active; the rest of the tree can be ignored (as far as parsing goes)


## Top-down parsing



Top-down parsing is the only parsing algorithm that always maintains a connected partial parse tree

## Bottom-up parsing



## Left-corner parsing




## Incremental parsing and NNs

+ Only the small fraction of nodes in the parse tree lying on the cut are active at any point in the string
- Can these be represented and updated by a recurrent NN? (Charniak)
- These nodes are manipulated with a LIFO stack discipline
- Unnatural in a NN architecture

